

Indoor SLAM Based on Composite Sensor Mixing Laser Scans and Omnidirectional Images

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Abstract—Vision sensors give mobile robots a relatively cheap means of obtaining rich 3D information of their environment, but lack the depth information that a laser range finder can provide. This paper describes a novel composite sensor approach that combines the information given by an omnidirectional camera and a laser range finder to efficiently solve the indoor Simultaneous Localization and Mapping problem and reconstruct a 3D representation of the environment. We report the results of validating our methodology using a mobile robot equipped with a 2D laser range finder and an omnidirectional camera.

I. INTRODUCTION

A key issue in mobile robotics is to give robots the ability to navigate in an autonomous way in unknown environments based only on their perception. Thus, a mobile robot must be equipped with a perception system capable of providing accurate information of its current location and its surroundings, so that the robot is able to reconstruct a reliable and consistent representation of the environment. There are two interdependent tasks that any mobile robot has to solve: localization and mapping. When neither the location of the robot nor the map are known, both tasks must be performed concurrently. This problem, known as Simultaneous Localization and Mapping (SLAM), has been largely studied since the seminal work of Smith and Cheeseman [18], [19], and is closely related to the development of sensor technology.

Nowadays, laser range finders have replaced sonars when possible because of its superior efficacy in estimating distances accurately and their better signal to noise ratio. Many techniques have been developed to make the most of this type of sensor for solving the SLAM problem. Since a laser scan directly provides metric information of the scene, the localization problem can be stated in terms of an odometry-based method where the incremental displacement is found by computing the best rigid transformation that matches two successive scans. To match two scans it is necessary to link the individual measurements in one scan with the corresponding measurements in the other scan. This association can be done either using an intermediate representation of the laser data (e.g. a polygonal approximation [1]) or directly, by exploiting the raw data [2].

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Several methods can be found in the literature for 2D and 3D scan matching. These methods are often categorized based on their association rule such as feature to feature or point to point matching. In the feature-based approach [3], [5], features such as line segments and corners are extracted from laser scans and then matched against each other. Such approach requires the identification of appropriate features in the environment. On the other hand, point to point matching does not require the environment to be structured or contain any predefined features.

The Iterative Closest Point [4] (ICP) algorithm is perhaps the most widely used point to point scan matching method that works with range sensors. ICP uses a nearest neighbor association rule to match points, and least squares optimization to compute the best transformation between two scans. Two enhanced methods based on ICP were proposed by Lu and Milius [6]: the Iterative Matching Range Point (IMRP) and the Iterative Dual Correspondence (IDC) method. Although ICP and its extensions are fast and in general produce good results, they are only guaranteed to converge towards a local minimum and may not always find the correct transformation. Furthermore, these algorithms suffer from computational complexity problems when dealing with large-scale environments because the point to point association rules they use result in a $O(n \log(n))$ complexity in the best case (where n is the number of points in a scan). To overpass these constraints, Diosi and Kleeman proposed the Polar Scan Matching method [7] which avoids searching for point associations by simply matching points with the same bearing. We will discuss this approach in more detail in Section III.

Despite all the work that has been done to improve techniques to use lasers to solve the SLAM problem, the use of 2D lasers alone limits SLAM to planar motion estimation and does not provide sufficiently rich information to reliably identify previously explored regions. Vision sensors are a natural alternative to 2D laser range finders because they provide richer perceptual information. Many works have pursued research on vision-based SLAM [9], either relying on feature-based representations [11] or, more recently, on a direct approach [10]. However, standard cameras only have a small field of view (typically between 30° and 40°) and can be easily affected by occlusion. In contrast,

using a catadioptric camera [21], [22] one can obtain a full 360° view of the environment. Image acquisition with these omnidirectional cameras has many advantages: it can be done in real time, it is easier to recognize previously observed places whatever the orientation of the robot is and it is also less likely that the robot gets *stuck* when facing a wall or an obstacle. Thus, vision sensors provide dense and rich 3D information about the environment. Nevertheless, vision alone does not provide the depth information that a laser range finder does, which is crucial for solving the localization problem.

In this paper we describe a hybrid sensor combining the advantages of a laser range finder and an omnidirectional camera. In its formulation, our work is close to Biber's [20]. The major difference is that the process we describe is fully automated and does not require manual postprocessing by an operator.

The rest of the paper is organized as follows: in Section II we describe the experimental testbed used to validate our methodology and discuss the data acquisition and synchronization process; Section III briefly overviews the Polar Scan Matching method, while our SLAM approach is presented in Section IV; in Section V we discuss the merging of omnidirectional images with laser range data to extract vertical lines and build a 3D representation of the environment; we end with some concluding remarks in Section VI.

II. EXPERIMENTAL TESTBED

Hannibal (Fig. 1) is our more recent robot from Neobotix mobile platform (MP-S500). Hannibal is equipped with a Sick LD-LRS1000 laser, capable of collecting full 360° data. The laser head revolves with variable frequency from 5Hz to 10Hz and the angular resolution can be adjusted up to 1.5° at multiples of 0.125° . The laser has a 30m range. To perform a 360° scan with a resolution of 0.25° , it was necessary to reduce the frequency of the rotor to 5Hz, thus obtaining 1400 data points per scan. The perspective camera is a progressive-scan CCD camera (Marlin F-131B) equipped with a telecentric lens and a parabolic mirror (S80 from Remote Reality). Careful calibration of the laser and the camera is required for merging image and laser data. We used the Matlab Omnidirectional Calibration Toolbox developed by Mei [13] to estimate the intrinsic parameters of the camera and the parameters of the parabolic mirror. For the calibration between the camera and the laser we used the method described in [12]. Figure 2 shows the projection of the laser range measurements on the omnidirectional image after calibration.

Data acquisition and synchronization. Odometry data arrives at a frequency of 50Hz, omnidirectional images at 10Hz and laser measurements at 5Hz. Since data from the different sensors that we use arrive at different frequencies, we implemented a function to synchronize the data as it comes out from the robot.

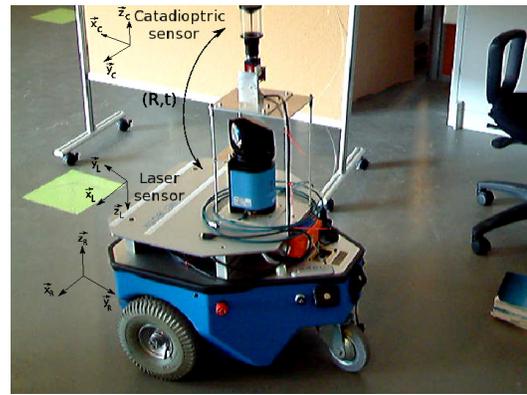


Fig. 1. Hannibal robot experimental testbed

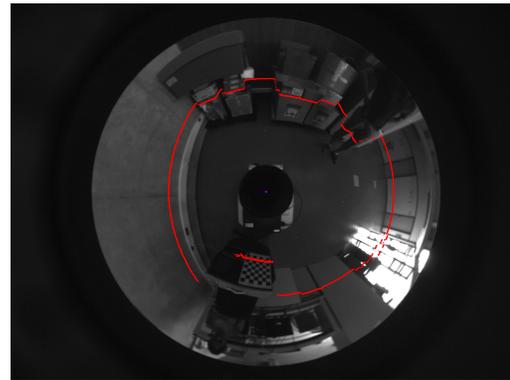


Fig. 2. Laser data projected on an omnidirectional image after calibration.

III. POLAR SCAN MATCHING

Polar Scan Matching (PSM) [7] is a point to point laser scan matching method that exploits the natural representation of laser scans in a polar coordinate system to reduce the complexity of the matching process. As other scan matching approaches, like the Iterative Closest Point (ICP) method, the PSM method finds the pose of a laser scan with respect to a reference scan by performing a gradient descent search for the transformation that minimizes the square error between corresponding points. In contrast to other matching methods, PSM avoids an expensive search for corresponding points by matching points with the same bearing. The method assumes the reference and current scans are given as sequences of range and bearing measurements of the form $\{r_{ri}, \phi_{ri}\}_{i=1}^n$ and $\{r_{ci}, \phi_{ci}\}_{i=1}^n$, respectively, and requires an initial estimate (x_c, y_c, θ_c) for the pose (position and orientation) of the current scan in the reference scan coordinate frame. The method may be best described by describing each of its phases:

a) Preprocessing: To remove outliers and increase the robustness of the method, the filter developed by A. Victorino in [23] is first applied to both scans. The measurements in each resulting scan are then classified into segments according to simple criteria: two consecutive measurements not further than a threshold or three measurements lying ap-

proximately on the same polar line are assigned to the same segment. Segments consisting of a single point are discarded (most mixed pixels). To aid the segmentation process, the maximum range is limited so that two consecutive readings belonging to the same segment cannot be too far apart.

b) Projection: To compute the error in the pose estimate of the current scan the method needs to know how the current scan would have been measured from the point of view of the reference scan. The projection of the current scan into the reference scan coordinate frame is a sequence of measurements $(r'_{ci}, \theta'_{ci})_{i=1}^n$ computed as follows

$$r'_{ci} = \sqrt{(r_{ci} \cos(\theta_c + \phi_{ci}) + x_c)^2 + (r_{ci} \sin(\theta_c + \phi_{ci}) + y_c)^2} \quad (1)$$

$$\theta'_{ci} = \text{atan2}(r_{ci} \sin(\theta_c + \phi_{ci}) + y_c, r_{ci} \cos(\theta_c + \phi_{ci}) + x_c) \quad (2)$$

The bearings of the above sequence do not necessarily coincide with bearings where the laser would have sampled a reading. A range measurement r'_{ci} is computed for each sample bearing by linear interpolation among points belonging to a same segment. Points that would have been occluded are not taken into account, only the smallest range measurement for a bearing is kept.

c) Translation and Orientation Estimation: The method alternates between translation and orientation estimation. After making a correction to the pose estimate, the projection phase is repeated with the corrected estimate. The process stops when the magnitude of the last position and orientation correction is smaller than a given threshold, hopefully indicating that a minimum has been reached. Translation is estimated using a standard weighted least squares method. A correction $(\Delta x_c, \Delta y_c)$ to the position estimate is found by minimizing the weighted sum of the square range residuals $\sum_{i=1}^n w_i (r_{ri} - r'_{ci})^2$ while leaving orientation unchanged. The weights are computed as recommended by Dudek and Jenkin [8],

$$w_i = \frac{c^2}{(r_{ri} - r'_{ci})^2 + c^2} \quad (3)$$

Orientation is estimated by computing the average range residual for 1° shifts of the current scan in a $\pm 20^\circ$ window. The new orientation estimate is found by fitting a parabola to the shift with the minimum average error and its left and right neighbors.

The implementation of the PSM method provided by Diosi is tailored to a laser with 1° angular resolution and 180° bearing range. These assumptions are used when transforming sample bearings from radians to indexes into arrays and back. We generalized Diosi's implementation to lift these assumptions. Our implementation is parametrized so that it can deal with lasers with arbitrary angular resolution and bearing range. In addition, instead of just returning the pose estimate at the moment the algorithm stops, our implementation keeps record of the estimate with the minimum error and returns it as a result.

IV. LOCAL AND GLOBAL MAPS WITH SLAM

$$\text{AFFINE-TRANSFORMATION}(x, y, \theta)$$

$$\text{return} \begin{bmatrix} \cos \theta & -\sin \theta & 0 & x \\ \sin \theta & \cos \theta & 0 & y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Fig. 3. Affine transformation for a translation (x, y) and a counterclockwise rotation around the origin by an angle θ .

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GLOBAL-MAP(scan[N])
1  SR ← scan[1]
2  T1 ← AFFINE-TRANSFORMATION(SR.x, SR.y, SR.θ)
3  T3 ← T1 × TL
4  Map ← APPLY-TRANSFORMATION(T3, SR)
5  for i ← 2 to N
6    SC ← scan[i]
7    T2 ← AFFINE-TRANSFORMATION(SC.x, SC.y, SC.θ)
8    T ← TL-1 × T1-1 × T2 × TL
9    (x, y, θ) ← (T(1,4), T(2,4), atan2(T(2,1), T(2,2)))
10   (x, y, θ) ← PSM(SR, SC, x, y, θ)
11   T'3 ← AFFINE-TRANSFORMATION(x, y, θ)
12   T3 ← T3 × T'3
13   Map ← Map ∪ APPLY-TRANSFORMATION(T3, SC)
14   SR ← SC
15   T1 ← T2
16  return Map

```

Fig. 4. Pseudocode of the procedure used to incrementally build a global map from a sequence of laser range scans with odometry information.

We build 2D local maps of the environment using the enhanced PSM implementation described in the previous section. Local maps will be used both, in the localization process and for mapping the environment. Later, these maps will be used in SLAM to reconstruct a 2D global map from which it is possible to recover the pose of the robot at each instant.

Let T_L be the rigid transformation between the laser coordinate frame and the robot coordinate frame. We fix as a global coordinate frame the coordinate frame of the odometry data. Let (x, y, θ) be the current position of the laser scan coordinate frame. The affine transformation matrix from the laser coordinate frame to the global coordinate frame is given by the procedure in Fig. 3.

We use the procedure in Fig. 4 to build a global map and reconstruct the path of the robot from a sequence of laser range scans with associated odometry data. We begin by taking the first scan in the sequence as the reference scan S_R . Initially, the map consists only of the points in the scan S_R represented in the global coordinate frame, but it will be incrementally enriched at each iteration of the loop. We keep at every moment a transformation matrix T_3 , from the coordinate frame of the laser in the reference scan frame to the global coordinate frame. At the beginning of each iteration we take the next scan in the sequence to update the current scan S_C . We then use the odometry data to obtain an initial estimate for the pose of the laser in the current scan with respect to the reference scan coordinate frame. We feed this estimate to the PSM procedure described in the previous

section, and get as a result a new estimate of the pose. Using this new estimate, we update the T_3 matrix, transform the points in the current scan to the global coordinate frame, and add them to the global map. The current scan becomes then the reference scan and the whole process is repeated again.

Because the short-term odometry of the robot when traveling on a flat surface is relatively accurate, in practice we do not need to use scan matching to compute the pose of the robot in every scan. Instead, we only use scan matching to get a better estimate of the pose of the robot when it has traveled a certain distance or rotated a certain angle, or when a certain lapse of time has passed since the last time scan matching was used.

Using the results obtained using the PSM algorithm, the odometry data of the whole sequence can be recomputed. It suffices to multiply after each iteration matrix T_3 by the transformation matrix T_L^{-1} , which gives the transformation matrix from the robot (not the laser) coordinate frame of the current scan to the global frame. The pose (x, y, θ) can be readily extracted from this last matrix. Figure 5 shows the position of the robot at several instants in the sequence as given by the original odometry data (in red) and as computed by SLAM (in green) superimposed on the generated map. The sequence was obtained by manually commanding the robot to explore the ground floor of a building in a closed loop. Note that although we did not perform closed-loop detection or corrections of any kind, the results are quite satisfactory. The recomputed odometry represents a big improvement over the original odometry that even drifts out of the building.

V. VERTICAL LINE EXTRACTION FROM OMNIDIRECTIONAL IMAGES AND LASER SCANS

This section explains the procedure we developed to extract vertical lines from omnidirectional images and to estimate their 3D positions using information from the laser range finder. We first project the laser information on the omnidirectional image in order to get an approximation of the depth information missing in the image. To achieve that, the unified projection model defined in [15] is applied, which is an extension of Geyer's [17] and Barreto's [16] models. The generalized camera projection matrix K is computed from the generalized focal lengths (γ_1, γ_2) and the principal point (u_0, v_0) :

$$K = \begin{bmatrix} \gamma_1 & 0 & u_0 \\ 0 & \gamma_2 & v_0 \\ 0 & 0 & 1 \end{bmatrix}$$

Using K , we can compute the normalized coordinates of a point \mathbf{p} in the image (represented in the camera coordinate frame) as $\mathbf{m} = [x, y, 1]^T = K^{-1}\mathbf{p}$. We then compute $\mathcal{X}_s = [X_s, Y_s, Z_s]$ as follows (see Fig 6):

$$\mathcal{X}_s = \begin{bmatrix} \frac{\xi + \sqrt{1 + (1 - \xi^2)(x^2 + y^2)}}{x^2 + y^2 + 1} x \\ \frac{\xi + \sqrt{1 + (1 - \xi^2)(x^2 + y^2)}}{x^2 + y^2 + 1} y \\ \frac{\xi + \sqrt{1 + (1 - \xi^2)(x^2 + y^2)}}{x^2 + y^2 + 1} - \xi \end{bmatrix}$$

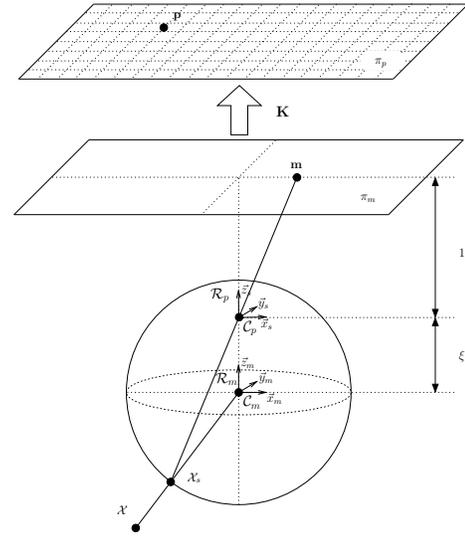


Fig. 6. Unified projection model

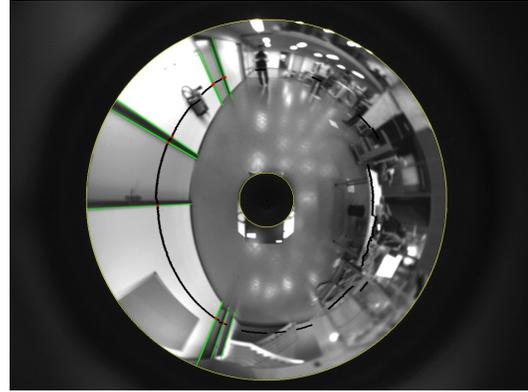


Fig. 7. Detection of vertical lines and the corresponding laser measurements.

where ξ is the mirror parameter, which is equal to 1 for parabolic mirrors.

We then extract the quasi-radial lines in the scene, corresponding to approximately vertical features (e.g. walls, facades, doors, windows). As we set the camera-mirror system perpendicular to the floor where the robot moves, we can guarantee that vertical lines are approximately mapped to radial lines on the camera image plane. To extract prominent vertical lines, we first apply the Canny edge detector to obtain a binary edge image and then apply the Hough transform to detect lines in the binary image. To extract vertical lines we compute the image center (i.e. where all radial lines intersect in) using a circle detector, and filter out the lines detected by the Hough transform that do not lie on radial directions.

As shown in Fig. 7, by overlapping in the omnidirectional image the laser scan data and the radial lines we can find the laser range measurements corresponding to vertical lines. This gives us the depth information missing. We detect those laser measurements and save them in the original camera frame together with its corresponding point in the image plane (which also corresponds to a point on a vertical line).



Fig. 5. Global map obtained by SLAM together with the original and recomputed position of the robot at several key instants.

Let $\mathbf{M}_0^s = [X_0^s, Y_0^s, 0]^T$ be a laser measurement lying on a vertical line expressed in the camera coordinate frame, Δ a 3D plane defined in the camera frame, and $\mathbf{m}_i^s = [x_i^s, y_i^s, z_i^s]^T$, $i = 1, 2$ the endpoints of the vertical line where the laser measurement lies expressed in the sphere (mirror) coordinate frame. These last points are computed by inverting the projections of the unified model of Fig. 6.

We reconstruct the 3D lines as follows. Let \mathbf{u}^s be the director vector. For every $\mathbf{M}_i^s \in \Delta$, the vector $\overrightarrow{\mathbf{M}_0^s \mathbf{M}_i^s}$ is colinear to \mathbf{u}^s . Thus,

$$\overrightarrow{\mathbf{M}_0^s \mathbf{M}_i^s} = \lambda_i \mathbf{u}^s \implies \begin{cases} X_i - X_0^s = \lambda_i u_x^s \\ Y_i - Y_0^s = \lambda_i u_y^s \\ Z_i - Z_0^s = \lambda_i u_z^s \end{cases} \quad (4)$$

$$\overrightarrow{\mathbf{O} \mathbf{M}_i^s} = \mu_i \overrightarrow{\mathbf{O} \mathbf{m}_i^s} \implies \begin{cases} X_i = \mu_i x_i^s \\ Y_i = \mu_i y_i^s \\ Z_i = \mu_i z_i^s \end{cases} \quad (5)$$

Substituting (5) in (4) we get the following system of equations

$$\begin{cases} \mu_i x_i^s - X_0^s = \lambda_i u_x^s \\ \mu_i y_i^s - Y_0^s = \lambda_i u_y^s \\ \mu_i z_i^s - Z_0^s = \lambda_i u_z^s \end{cases} \quad (6)$$

If Δ is a vertical plane in the sphere frame R_s , i.e. $\mathbf{u}^s = [0, 0, 1]^T$, then:

$$\begin{cases} \mu_i x_i^s - X_0^s = 0 \\ \mu_i y_i^s - Y_0^s = 0 \\ \mu_i z_i^s - Z_0^s = \lambda_i \end{cases} \quad (7)$$

Because we know $[x_i^s, y_i^s, z_i^s]^T$ and $[X_0^s, Y_0^s, 0]^T$, we can compute μ_i for each i . We can then substitute in Equation (5) to obtain the extreme points of the lines in Δ . Finally, we apply the homogeneous transformation to transform the coordinates of those points to the global coordinate system and trace the 3D lines. The result is shown in Fig. 8. Observe how the vertical lines are consistent with the 2D map.

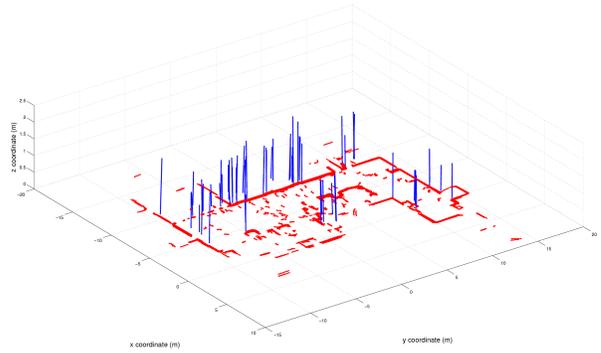


Fig. 8. Environment with 3D lines

VI. DISCUSSION AND PERSPECTIVES

This paper describes an original composite sensor approach that takes advantage of the information given by an omnidirectional camera and a laser range finder to efficiently solve the Simultaneous Localization and Mapping problem for indoor environments, and to reconstruct a 3D representation of the environment. The accompanying video illustrates the incremental generation of a 2D map and the estimation of the robot trajectory alongside the laser range data projected on omnidirectional images. It also shows the vertical lines detected in the images and their mapping into a 3D reconstruction of the environment.

In order to show the robustness of the methodology, we tested the algorithm with a sequence taken in a different indoor environment with our old robot Anis which is equipped with the same catadioptric camera and an AccuRange 4000 2D laser range finder. This laser is composed of a laser telemeter with a rotating mirror that allows measurements of points on 360°, except for an occlusion cone of approximately 30° caused by the assembly of the mirror. The resulting 2D map is shown in Figure 9. The vertical line extraction and the reconstruction of the 3D environment were

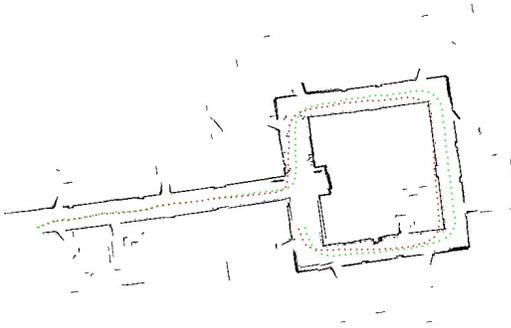


Fig. 9. Global Map obtained by SLAM in Borel Building

consistent as well.

The SLAM problem has been solved using many different approaches, however some important problems need to be addressed that are often directly linked to the sensors used. Laser range finders cannot help in evaluating the translation of a robot moving in a straight line in a corridor. Mapping in dynamic environments is also hard with only laser data. On the other hand, using visual sensors alone introduces issues such as propagating correctly the scale factor, initializing the range when using a monocular sensor, and merging data when using multiples cameras.

In our approach, the laser provides metric information of the environment that helps to fix a scale factor (removing the difficulty of propagating the scale factor) without the need to use multiple cameras. Throughout the paper we have identified several advantages of combining laser and visual sensors. Our experimental results are encouraging and give us valuable insight into the possibilities offered by this composite sensor approach.

We have considered several research directions that could be pursued to improve the results obtained so far. We have thought about extending our algorithm with loop closure detection. This would allow the algorithm to detect previously visited locations and improve the accuracy of mapping and the precision in the estimation of the robot pose. Being able to detect previously visited places is of great importance to solve the problem of global localization and to recover the robot from kidnapping, a situation occurring when the robot is displaced by something out of its control (e.g. taking an elevator, being transported from one location to another). Therefore, solving the loop closure problem will not only improve SLAM performance, but will as well enable new capabilities.

Further work will concentrate on an extension of the PSM algorithm to exploit the information about vertical lines detected using omnidirectional images. Segmentation of the ground (floor) to extract planes on the image would allow a dense (textured) 3D reconstruction by warping the images onto the geometric model of the world. Finally, we believe the general approach can be extended to solve the full six degrees of freedom (6DOF) SLAM problem, which is an

active field of research.

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